Mathematical Foundations of NAMS: A Real-Time Attention Metric for Urban Environments

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Abstract

This whitepaper introduces NAMS: North² Attention Metric Score, a computational approach for quantifying attention. The methodology leverages geometric modeling and spatial analysis to determine focus direction, allowing real-time estimation of attention in urban environments. Applications span various fields such as measuring human engagement and adaptive content delivery. We outline the underlying mathematical framework, implementation strategy, and potential commercial use cases.

1 Introduction

Understanding human attention in dynamic environments is essential for a range of applications, e.g. intelligent interfaces and engagement analytics. This whitepaper presents a framework that utilizes real-time computational modeling to estimate attention distribution with high temporal and spatial resolution. Using geometric transformations and probabilistic inference, our method delivers a continuous attention metric score that performs in diverse settings.

2 Mathematical Framework

2.1 Geometric Representation of Attention

To represent attention in a structured manner, we define a spatial coordinate system where an individual's focus is modeled as a vector $\mathbf{A}$ originating from a reference point P and extending in the estimated direction of interest. Given a fixed reference frame, the attention vector is described as:

$$\mathbf{A} = \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} = R(\theta, \phi)\mathbf{P} + \mathbf{T}, \tag{1}$$

where $\mathbf{P}$ is a reference position in 3D space, $R(\theta, \phi)$ is a rotation matrix parameterized by angular estimates (θ, ϕ), and $\mathbf{T}$ is a translational offset capturing local positional adjustments.

2.2 Attention Probability Distribution

To create a robust real-time attention measurement, we define an attention probability distribution over time. At any sampling interval t, the likelihood of an individual focusing on a specific region Ω is modeled as:

$$P(A_t \in \Omega) = \int_{\Omega} f(A_t)\, dA_t, \tag{2}$$

where $f(A_t)$ is a probability density function derived from past observations and adaptive weighting factors.

2.3 Statistical Sampling and Filtering

Given the real-time nature of attention estimation, a sequential sampling approach is required. Using a Bayesian update model, we refine the attention estimate iteratively:

$$P(A_t|A_{t-1}, D_t) \propto P(D_t|A_t)P(A_{t-1}), \tag{3}$$

where: A_t is the estimated attention state at time t, D_t represents new observational data, $P(D_t|A_t)$ accounts for measurement uncertainty, and $P(A_{t-1})$ serves as the prior distribution from previous time steps.

For enhanced stability, we employ a Kalman filter or a particle filtering technique to smooth rapid fluctuations in attention direction.

2.4 Context-Specific Normalization

NAMS is designed to be adaptable to different operational contexts, ensuring consistent measurement across varying environments. Depending on the setup, different normalization techniques are used, particularly when considering static versus dynamic configurations. In scenarios with a static setup (e.g., in a fixed location / controlled area), the normalization accounts for environmental constraints and expected attention distributions. A baseline distribution A_{baseline} is first established by averaging the historical attention value. The normalized score is then computed as:

$$NAMS_{\text{static}} = \frac{A - A_{\text{baseline}}}{A_{\max} - A_{\min}} \tag{4}$$

where is the raw attention score at a given moment, A_{baseline} is the baseline attention level from past observations, and $A_{\max}$, $A_{\min}$ are the observed extreme attention values.

This normalization ensures that variations in attention are evaluated relative to the typical patterns observed in that environment.

When measuring attention with a dynamic setup (e.g., deployed in a moving environment / dynamically changing conditions), the normalization is designed to be adaptive. Instead of a static baseline, we employ a rolling normalization approach, where normalization parameters are updated continuously as follows:

$$NAMS_{\text{dynamic}} = \frac{A - \mu_t}{\sigma_t} \tag{5}$$

where μ_t is the dynamically computed mean of attention values over a recent time window, and σ_t is the standard deviation over the same window.

To enhance the stability of the attention metric beyond standard normalization, a Gaussian kernel could be employed. The following kernel function applies a weighted transformation to the raw attention scores, creating a more robust and temporally stable representation:

$$NAMS_{\text{smoothed}} = \frac{A_t \cdot e^{-\frac{(A_t-\mu_t)^2}{2\sigma_t^2}}}{Z} \tag{6}$$

where A_t denotes the instantaneous attention score at time t,μ_t represents the mean attention score computed over a time window, σ_t corresponds to the standard deviation of attention scores within the same window, and Z is a normalization constant.

2.5 Adaptive Scaling Across Environments

For NAMS to be universally applicable, an adaptive scaling factor S can be introduced to compensate for variations across different sensor configurations. The final normalized attention score can be adjusted using:

$$NAMS_{\text{final}} = S \cdot NAMS_{\text{context}} \tag{7}$$

where S is a context-specific scaling coefficient that ensures comparability between different measurement setups.

This adaptive approach allows NAMS to maintain consistency across various deployment conditions, whether in controlled laboratory settings, urban environments, networked locations or highly dynamic applications.

3 Implementation Strategy

3.1 Real-Time Processing Pipeline

NAMS operates through a structured pipeline designed to continuously analyze the attention metric score. Data are collected at predefined intervals at an arbitrary frequency, ensuring that relevant information is consistently captured. From this input, geometric features are extracted to define an attention vector that represents engagement levels. As new data is processed, probabilistic updates refine the real-time attention score, dynamically adapting in case of changing conditions. Finally, aggregated data undergoes statistical analysis to identify robust patterns, completing the attention estimation.

3.2 Adaptive Sampling at Arbitrary Intervals

Unlike a fixed-interval attention tracking system, our framework allows for flexible sampling intervals. By dynamically adjusting the sampling rate f_s based on observed variance σ_A^2, the system optimizes data processing efficiency. The adaptive interval selection follows:

$$\Delta t = \frac{k}{1+\sigma_A^2} \tag{8}$$

where k is a scaling parameter balancing responsiveness and computational load.

4 Commercial Applications

4.1 Engagement and Attention

In urban environments, real-time attention analysis enables A/B-testing of content, dynamic content adaptation and delivery. This is particularly useful in retail and digital advertising.

4.2 Human-Machine Interaction

Attention-aware interfaces improve usability by responding dynamically to focus. Applications include adaptive interface elements in urban environments, smart displays, and other automated systems.

5 Conclusion

This whitepaper presents a mathematical and statistical framework for real-time attention estimation. Using geometric transformations, probabilistic modeling, and adaptive sampling, our approach delivers robust attention metrics in various environments. The methodology's flexibility allows for commercial deployment in e.g. engagement analytics and intelligent interface design.